Autumn 2023

STAT 7301 Advanced Statistical Theory I

lecture:	MWF 9:10–10:05 in Cockins Hall 232
instructor:	Vincent Q. Vu (vqv at stat osu edu)
office:	Cockins Hall 428B
office hours:	Thursdays, 2:00pm—2:55pm on Zoom
web:	Class schedule, assignments, and course announcements will be posted on CarmenCanvas (carmen.osu.edu)
prerequisites:	STAT 6802, or permission of the instructor

1 Overview

Statistics 7301 is a course on the fundamentals of statistical theory and is intended for second-year Ph.D. students in statistics. The course is based in part on chapters 2–4,8–9 of the required book *Theoretical Statistics: Topics for a Core Course*, chapters 2 and 6 of the required book *All of Nonparametric Statistics*, and notes provided by the instructor. The topics of the course include:

1. Fundamentals
    - Statistics, sufficiency, and completeness
    - Exponential families
    - Rao-Blackwell theorem
    - Fisher information
2. Methods of estimation
    - Unbiased estimation
    - Maximum likelihood
    - Minimum contrast estimation
3. Asymptotic approximations (a.k.a. large sample theory)
    - Consistency
    - Delta method
    - Asymptotic normality and efficiency
4. Nonparametric estimation
    - Estimating the CDF and statistical functionals
    - Influence functions and nonparametric Delta method
    - Density estimation

Although there will be overlap in the topics covered in STAT 7301 and STAT 6801/6802, some of the most important differences between this class and the 680x classes is that STAT 7301

- presents the theory in greater depth and detail;
- involves mathematically rigorous proofs;
- requires more sophisticated mathematical analysis.

2 Textbook

The in-class lectures and notes are the canonical source for the course. The following books are also required for supplemental reading and assignments:

- Keener, R.: *Theoretical Statistics: Topics for a Core Course*. Springer.
  - htthttps://link.springer.com/book/10.1007/978-0-387-93839-4
- Wasserman, L.: *All of Nonparametric Statistics*. Springer.
  - https://link.springer.com/book/10.1007/0-387-30623-4

The notation and nomenclature used in lecture and the depth of coverage of material will occasionally deviate from these books. The instructor will try to alert students to these differences, but ultimately students are expected to pay attention to these differences themselves.

The follows books are optional references:

- Bickel, P. J. and Doksum, K. A.: *Mathematical Statistics: Basic Ideas and Selected Topics, Vol. 1*. CRC Press.
- van der Vaart, A. W.: *Asymptotic Statistics*. Cambridge University Press.
- Lehman, E. L. and Casella, G.: *Theory of Point Estimation*, second edition. Springer.
  - https://link.springer.com/book/10.1007/b98854

3 Prerequisites

Statistics 6802, or permission of the instructor. Mathematical analysis and probability theory are the primary tools of statistical theory. **Students are expected to be able to read and write rigorous mathematical proofs.**

4 Coursework & Grading

There will be homework assignments, two in-class exams, and a final exam. Your grade will be based on the following:

- 25% Homework (9 assignments)
- 25% Midterm Exam 1 (October 6)
- 25% Midterm Exam 2 (November 8)
- 25% Final Exam (December 11, 10:00 – 11:45)

Inform the instructor of any scheduling conflicts at least three weeks in advance.

4.1 In-class participation

I expect students to attend each class and participate in the discussion, for example, by asking and answering questions. I may occasionally take attendance. If you must miss a class, please kindly let me know in advance.

4.2 Homework

Homeworks will generally be assigned on a weekly basis and are due in class on the due date. If you cannot attend class on the due date of the homework, then ask a classmate to submit the homework. I will **not accept homework by email**.

I may reuse problems from other or similar courses that have been taught in the past. **Do not copy or look at previous solutions while preparing your answers**.

While it is acceptable to collaborate with other students to solve homework problems, each student must write up their own solutions. If you do collaborate, then you must indicate on your submission the students that you collaborated with.

In general, late homework will not be accepted. In the case of an emergency (medical, family, etc…), please let me know as soon as possible so that I can work with you to give a reasonable extension.

4.3 Exams

All exams are closed book. No make-up exams will be given. The in-class exams will largely be based on the content of the homeworks preceding them. The final exam will be cumulative but will emphasize more recent material.

5 Additional information and policies

5.1 Academic Misconduct

It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentlife.osu.edu/csc/).

5.2 Disability Services

The university strives to maintain a healthy and accessible environment to support student learning in and out of the classroom. If you anticipate or experience academic barriers based on your disability (including mental health, chronic, or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services (https://slds.osu.edu). After

registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion.

5.3 Religious Accommodations

It is Ohio State's policy to reasonably accommodate the sincerely held religious beliefs and practices of all students. The policy permits a student to be absent for up to three days each academic semester for reasons of faith or religious or spiritual belief.

Students planning to use religious beliefs or practices accommodations for course requirements must inform the instructor in writing no later than 14 days after the course begins. The instructor is then responsible for scheduling an alternative time and date for the course requirement, which may be before or after the original time and date of the course requirement. These alternative accommodations will remain confidential. It is the student's responsibility to ensure that all course assignments are completed.

6 Schedule

This schedule is subject to revision. Students are expected to attend class meetings and to regularly check CarmenCanvas for updates.

week	date	#	topic	assignment
1	8/23	1	Statistical models	
1	8/25	2	Decision theory	
2	8/28	3	Sufficiency	
2	8/30	4	Minimal sufficiency	
2	9/01	5	Exponential families	
3	9/04		**No class** (Labor day)	
3	9/06	6	Exponential families	
3	9/08	7	Exponential families	HW 1
4	9/11	8	Exponential families	
4	9/13	9	Convex loss functions; Jensen's inequality	
4	9/15	10	Rao–Blackwell Theorem	HW 2
5	9/18	11	Completeness and ancillarity	
5	9/20	12	Unbiased estimation	
5	9/22	13	Fisher information	HW 3
6	9/25	14	Information inequality	
6	9/27	15	Method of moments	
6	9/29	16	Maximum likelihood	HW 4
7	10/02	17	MLE in exponential families: theory	
7	10/04	18	MLE in exponential families: examples	
7	10/06		**Exam 1**	Exam 1
8	10/09	19	M-estimation	
8	10/11	20	Overview of asymptotics	

week	date	#	topic	assignment
8	10/13		**No class** (Autumn break)	
9	10/16	21	Consistency of MLE in exponential families	
9	10/18	22	Consistency of M-estimators	
9	10/20	23	Consistency of M-estimators (ULLN)	HW 5
10	10/23	24	Consistency of M-estimators and MLE	
10	10/25	25	Delta method	
10	10/27	26	Delta method	HW 6
11	10/30	27	Asymptotic normality of Z-estimators	
11	11/01	28	Asymptotic normality and efficiency of MLE	
11	11/03	29	Nonparametric estimation and the empirical CDF	HW 7
12	11/06	30	Statistical functionals	
12	11/08		**Exam 2**	Exam 2
12	11/10		**No class** (Veterans day)	
13	11/13	31	Influence functions	
13	11/15	32	Functional Delta method	
13	11/17	33	Nonparametric density estimation	
14	11/20	34	Asymptotic MISE of the histogram	HW 8
14	11/22		**No class** (Thanksgiving)	
14	11/24		**No class** (Indigenous Peoples' Day)	
15	11/27	35	Kernel density estimation	
15	11/29	36	MISE bound for kernel density estimation	
15	12/01	37	TBD catch-up	
16	12/04	38	TBD catch-up/review	
16	12/06	39	TBD catch-up/review	HW 9
	12/11		**Final Exam** (10:00 – 11:45)	Final Exam